Modern Approach

Key points

- Modern asset pricing assumes the existence of a trade-off between risk and returns and that only systematic risk factors should enter pricing equations.
- Single factor models like the CAPM, while widely used for private asset pricing are not robust and fail to capture risk and return dynamics.
- Multi-factor models offer a more suitable approach. They posit the existence of several priced risk factors, the aggregation of which explain the risk premia required by investors.
- The risk factor exposures (or loadings) of individual asset are typically denoted as betas (β) and risk factor prices or premia as lambdas (λ).

While the measurement of risk is not an objective of the fair-value framework, from the point of view of finance theory the **starting point of any asset-pricing approach is to postulate the existence of a trade-off between risk and returns.** From this initial postulate, the main question is to determine the appropriate measure of risk, from which a corresponding measure of expected returns, that is, the appropriate, risk-adjusted discount rate, can be derived.

A second tenet of modern finance theory is that idiosyncratic or company-specific risks cancel each other out in a well-diversified portfolio, and therefore **only systematic risks should enter into pricing equations.**

Hence, academic finance provides **a framework for estimating investors' expected rates of return given common sources of risk found in financial assets**. Expected returns predicted by statistically robust risk-factor models, taking into account current market conditions, provide a basis for the 'fair' discount rate that should prevail in the principal market.

Below, we briefly discuss standard factor models of expected returns, academic guidance to determine which factors should be used in such models, how they are estimated for assets traded in liquid markets where times series of prices and returns are readily available, and how this approach may be applied to seldom-traded private assets like unlisted infrastructure.

In other, we also discuss the relevance of risk factors identified in the academic literature to the valuation of infrastructure companies, and the statistical approach taken to estimate the value of factor prices in unlisted infrastructure.

Models of Expected Returns

The simplest model of expected returns is the well-known capital asset pricing model (CAPM)[1][2], according to which higher returns can only be achieved by exposing a well-diversified portfolio to higher market risk, also known as a higher market *beta*. This model serves as the basis for the discount-rate formula described in equation (1).

Technically, the market *beta* is the coefficient of a single independent variable (the market return) in an ordinary least-square (OLS) regression explaining the variance of excess returns for a single asset (the dependent variable).

(1) $$E(R_i) - R_f = \beta_i \left(E(R_m) - R_f \right) + \epsilon_i$$

with R_m as the market return, R_f the risk-free rate, and ϵ_i the specific risk of investment i, which by definition is not correlated with R_m.

The CAPM implies that the 'market index' to which any asset relates is 'mean-variance efficient', that is, fully captures the asset's potential exposure to all systematic sources of risk. Empirically this prediction is not robust and has been proven inaccurate by multiple studies for listed equities[3].

Multi-factor models have been developed to address the lack of statistical robustness of the single-factor CAPM. Important work by Fama and French established multi-factor models of asset returns as standard tools to create measures of expected returns.

Well-known industry versions of these ideas that relate expected returns in the next period ($t+1$) to current exposures to risk factors known today (at time t) include the BARRA® multi-factor models and can be written (omitting the t time subscript for simplicity):

(2) $$E(R_i) - R_f = \beta_{i,1} F_1 + \cdots + \beta_{i,K} F_K + \epsilon_i$$

where $\beta_{i,k} = \rho_{i,k} \frac{\sigma_i}{\sigma_k}$ is the factor exposure or *loading* of asset i to factor k (with $\rho_{i,k}$ being the correlation between asset and factor returns, and σ_i and σ_k being the volatility of asset and factor returns, respectively); F_k is the return to factor k during the period from time t to time $t+1$; and ϵ_i is the asset's specific or idiosyncratic return during that period that cannot be explained by the K factors.

The K factors used in such models can include industrial and geographic segmentations of the data or various economic mechanisms that can be expected to have a systematic effect on average returns such as the tendency of well-performing firms to continue to perform (the so-called momentum factor), as well as factors sometimes identified as 'anomalies' (like the outperformance of a 'low volatility' factor).

Risk-Factor Pricing

Beyond single- or multi-factor CAPMs, the second building block of modern asset pricing is the arbitrage pricing theory (APT)[4]. Under arbitrage pricing, pure arbitrage profits are not possible and differences between expected returns (or prices) and actual returns are the result of individual assets' relative exposure (betas) to a combination of zero-mean risk factors (or 'surprises') f_k. Hence,

(3) $$R_i = E(R_i) + \beta_{i,1} f_1 + \cdots + \beta_{i,K} f_K + \epsilon_i = E(R_i) + \sum_{k=1}^{K} \beta_{i,k} f_k + \epsilon_i$$

where $E(R_i) = R_f + \sum_k \beta_{i,k} E(F_k)$.

All available information at the time of pricing is already incorporated in expected returns and the impact of factors on returns *ex post* is the combination of individual assets' exposure to these factors (the $\beta_{i,k}$) and any unexpected change or 'surprise' in the realisation of the f_k, which have an expected value of zero.

Here again, expected excess returns are written as the linear combination of multiple risk-factor exposures and their risk premia or prices on the measurement date, that is,

$$E(R_i) - R_f = \lambda_1 \tilde{\beta}_{i,1} + \cdots + \lambda_K \beta_{i,K}$$

where λ_j is the price of risk or premium of the j^{th} risk factor with $j = 1 \ldots K$, and $\tilde{\beta}_{i,j} = \beta_{i,j}\sigma_j$ is the re-defined measure of asset's exposure to the risk factor, which is equal to $\beta_{i,j}$ if factors are standardised to have a standard deviation of 1.

That is, expected (excess) returns are the sum of the risk-factor exposures $\tilde{\beta}_{i,j}$ of asset i at time t times the respective price of each risk factor λ_j to which asset i is exposed. Realised excess returns are thus written:

$$R_i - R_f = \sum_{k=1}^{K} \tilde{\beta}_{i,k}(f_k + \lambda_k) + \epsilon_i$$

Thus, realised excess return for asset i are generated by its exposure to risk factors and their unexpected realisation, as well as firm-specific risk. Because we will work only with standardised factors, there is no distinction between $\tilde{\beta}$ and β, and for notational convenience we will only use β to write expected returns, so that:

$$R_i - R_f = \sum_{k=1}^{K} \beta_{i,k}(f_k + \lambda_k) + \epsilon_i$$

where $\beta_{i,k} = \rho_{i,k}\sigma_k$.

The price of asset i today should thus equal the sum of all future cash flows discounted at the APT rate defined by equation(3), in which expected returns for asset i are a linear function of various factors and the sensitivity to changes in each factor, as represented by each asset-specific beta coefficient.

Multi-factor models focus on finding robust and persistent statistical effects that can explain and predict the price preferences of buyers and sellers over time. Modeling discount factors reflecting the fair value of financial assets is thus a matter of identifying relevant priced risk factors and determining both individual assets' exposure to each factor as well as the market price or required premia of a pure exposure to each such factor taken independently.

In effect, CAPM and APT are the only two theoretical frameworks that provide a solid foundation for computing the trade-off between risk and returns [5]. APT in particular leaves the identification of the relevant factors open and justifies the need to explicitly identify and test the factors impacting returns for different types of financial assets.

References

1. 1
2. 1
3. 1
4. 1
5. 1